TO: Next Year's Calculus Students
FROM: R. Garcia, AP Calculus BC Teacher

As you probably know, the students who take AP Calculus AB and pass the Advanced Placement Test will place out of one semester of college Calculus; those who take AP Calculus BC and pass the Advanced Placement Test will place out of two semesters of college Calculus, depending on your score and where you attend college. Because Calculus BC covers two semesters, you will be learning Calculus faster than the students who take Calculus AB. In order to have enough time to learn the material of two college-level courses, we will start learning Calculus as soon as possible when school begins and will not be able to spend time reviewing the material you learned in Algebra, Geometry, and Precalculus.

Attached is a summer homework packet, which will be due the first day of Calculus class in August. The material in the packet should be material you learned in Algebra II and Precalculus.

<u>We will discuss the packet the first week of Calculus class</u>. <u>During the first week of school, you will take a test on the material in the packet</u>.

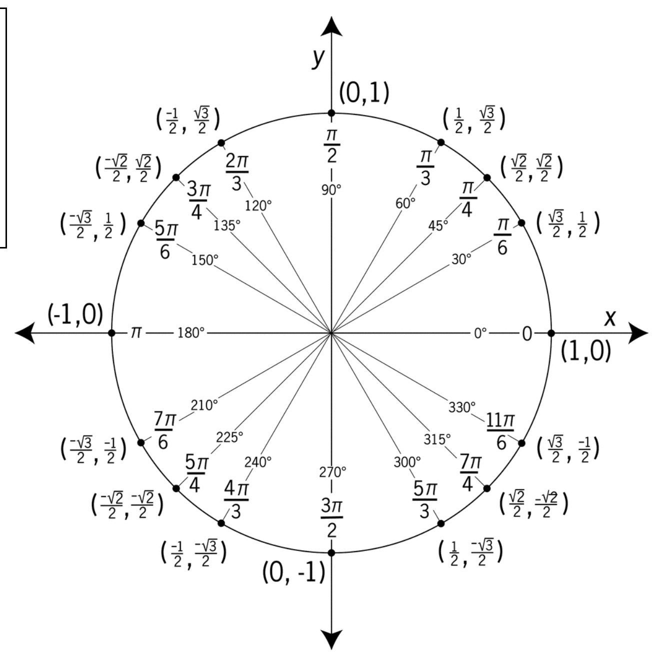
My recommendation is that you look over the problems in the packet when you receive it and work the limit and derivative questions now while it is fresh in your mind. Wait until the week before school starts to work the remaining problems and review the others. <u>*This packet will take longer than you think, be smart with your time and* ***DO NOT WAIT UNTIL THE DAY BEFORE SCHOOL STARTS TO BEGIN IT!!***</u>

In class, we will be using the **TI-nspire CX CAS**. I will have a class set available during class, but it is highly recommended that you have your own to use at home and become familiar with. This calculator is almost the same as the non-CAS version, but much more powerful and you can learn a lot by using it correctly. If you have the non-CAS version, you will be fine as well, but I really recommend the CAS version. Shop around and see if you can find a good deal. Note: The AP Calculus test has a calculator section.

I am looking forward to seeing you in Calculus in August.

Trig Identities You Should Know:

$\sin^2\theta+\cos^2\theta=1$	$\sin(2\theta)=2\sin\theta\cos\theta$
$1+\tan^2\theta=\sec^2\theta$	$\cos(2\theta)=\cos^2\theta-\sin^2\theta$
$1+\cot^2\theta=\csc^2\theta$	$\cos(2\theta)=1-2\sin^2\theta$
	$\cos(2\theta)=2\cos^2\theta-1$



CALCULUS

SUMMER HOMEWORK

<u>This homework packet is due the beginning of the first week of school.</u> <u>You will take a test on the material in the packet near the beginning of school.</u>

Work these problems on <u>notebook paper</u>. <u>All work must be shown</u>.
Use your graphing calculator only on problems <u>44 - 62</u>.

Find the x- and y-intercepts and the domain and range, and sketch the graph. <u>No calculator</u>.

1. $y=\sqrt{x-1}$
2. $y=\sqrt{9-x^2}$
3. $y=\dfrac{|x|}{x}$
4. $y=\sin x,\ -2\pi\le x\le 2\pi$
5. $y=\cos x,\ -2\pi\le x\le 2\pi$
6. $y=\tan x,\ -2\pi\le x\le 2\pi$
7. $y=\cot x,\ -2\pi\le x\le 2\pi$
8. $y=\sec x,\ -2\pi\le x\le 2\pi$
9. $y=\csc x,\ -2\pi\le x\le 2\pi$
10. $y=e^x$
11. $y=\ln x$
12. $y=\begin{cases}-1, & \text{if } x\le -1\\ 3x+2, & \text{if } |x|<1\\ 7-2x, & \text{if } x\ge 1\end{cases}$
13. $y=\begin{cases}x^2+1, & \text{if } x>0\\ -2x+2, & \text{if } x\le 0\end{cases}$

Find the asymptotes (horizontal, vertical, and slant), symmetry (x-axis symmetry, y-axis symmetry, or origin symmetry), and intercepts, and sketch the graph.
<u>No calculator</u>.

14. $y=\dfrac{1}{x-1}$
15. $y=\dfrac{1}{(x+2)^2}$
16. $y=\dfrac{2(x^2-9)}{x^2-4}$
17. $y=\dfrac{x^2-2x+4}{x-1}$

Use a number line graph to solve. <u>No calculator</u>.

18. $x^2-x-12>0$
19. $(x-2)^2(x+1)^3(x-5)\le 0$
20. $\dfrac{3x-2}{x+4}\le 0$
21. $\dfrac{(2x+5)(x-1)^2}{(x+2)^3}\ge 0$

Evaluate. <u>No calculator</u>.

22. $\cos\dfrac{5\pi}{6}$
23. $\sin\dfrac{3\pi}{2}$
24. $\tan\dfrac{5\pi}{4}$
25. $\sin\dfrac{7\pi}{4}$
26. $\cos\pi$
27. $\tan\dfrac{2\pi}{3}$
28. $\sec\dfrac{4\pi}{3}$
29. $\csc\dfrac{\pi}{4}$
30. $\cot\dfrac{2\pi}{3}$

Evaluate. <u>No calculator</u>.

31. $\tan\left(\operatorname{Cos}^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right)$
32. $\sec\left(\operatorname{Arc\,sin}\left(-\dfrac{\sqrt{2}}{2}\right)\right)$
33. $\cos\left(\operatorname{Sin}^{-1}(2x)\right)$
34. $\sec\left(\operatorname{Arc\,tan}(4x)\right)$

Solve. Give exact answers in radians, $0 \le x \le 2\pi$. <u>No calculator</u>.

35. $2\cos^2 x + 3\cos x - 2 = 0$

36. $2\sin^2 x - \cos x = 1$

37. $\sin(2x) = \cos x$

38. $2\cos(2x) + 1 = 0$

39. $2\csc^2 x + 3\csc x - 2 = 0$

40. $\tan^2 x - \sec x = 1$

41. $2\cos\left(\frac{x}{3}\right) - \sqrt{3} = 0$

42. $\tan(2x) = -\sqrt{3}$

43. $2\sin(3x) - \sqrt{3} = 0$

Solve. Show all steps. Use your **<u>calculator</u>**, and give decimal answers correct to **<u>three</u>** decimal places.

44. $e^{2x+3} = 37$

45. $e^{2x} - 5e^x + 6 = 0$

46. $e^x - 12e^{-x} - 1 = 0$

47. $\frac{50}{4 + e^{2x}} = 11$

48. $\log_4\left(x^2 - 3x\right) = 1$

49. $\ln(5x - 1) = 3$

50. $\log_2(x+3) + \log_2(x-1) = \log_2 12$

51. $\log_8(x+5) - \log_8(x-2) = 1$

52. $\log_6\left(\log_4\left(\log_2 x\right)\right) = 0$

53. $\log_3(\log_2(\log_5 25)) = x$

54. The number of students in a school infected with the flu t days after exposure is modeled by the function $P(t) = \frac{300}{1 + e^{4-t}}$.

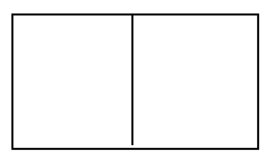
(a) How many students were infected after three days?
(b) When will 100 students be infected?

55. Exponential growth is modeled by the function $n = n_0 e^{kt}$. A culture contains 500 bacteria when $t = 0$. After an hour, the number of bacteria is 1200.

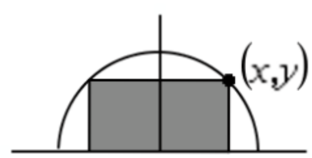
(a) How many bacteria are there after four hours?
(b) After how many hours will there be 8000 bacteria?

Write a function for each problem, and use your graphing **<u>calculator</u>** to solve. Give decimal answers correct to <u>three</u> decimal places. Be sure to sketch the graph you used, and label it with your answer.

56. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals, as shown. What dimensions should be used so that the enclosed area will be a maximum?



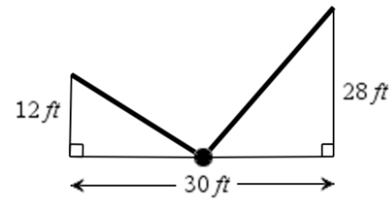
57. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{25 - x^2}$ as shown. What length and width should the rectangle have so that its area is a maximum?



58. Four feet of wire is to be used to form a square and a circle. Find the length of the sides of the square and the radius of the circle that will enclose the maximum total area?

59. The sum of the perimeters of an equilateral triangle and a square is 10. Find the dimensions of the triangle and the square that produce a minimum total area.

60. Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be secured by two wires, attached to a single stake, running from ground level to the top of each post. How far from the left post should the stake be placed to use the least wire?



61. A manufacturer makes a metal can in the shape of a cylinder that holds $1000\,\text{cm}^3$ of oil. What radius and height will minimize the amount of metal in the can?

62. A cylindrical metal container, open at the top, is to have a capacity of $24\pi\,\text{in}^3$. The cost of material used for the bottom of the container is \$0.15 per sq. in., and the cost of the material used for the curved part is \$0.05 per sq. in. Find the dimensions that will minimize the cost of the material, and find the minimum cost.

<u>No calculator</u>.

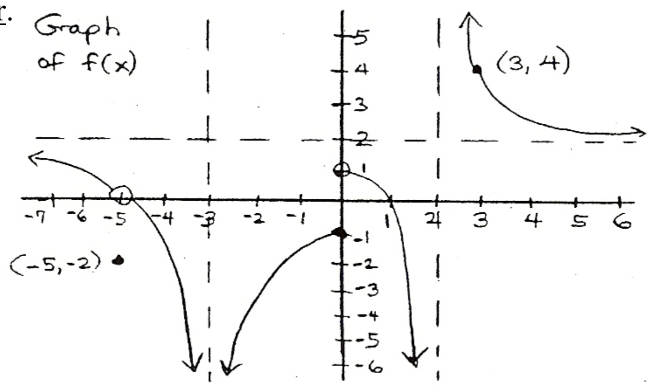
63. Convert the equation $2x-3y=8$ into polar form.

64. Convert the equation $r^2-2r\sin\theta=0$ into rectangular form

Make a table, and sketch the graph.

65. $r=3\sin\theta$	66. $r=2+2\cos\theta$	67. $r=1+2\sin\theta$
68. $r=3-2\cos\theta$	69. $r=4\cos(3\theta)$	70. $r=3\sin(2\theta)$

Use the figure to find the limit. <u>No calculator</u>.



71. $\lim\limits_{x\to 3} f(x)$

72. $\lim\limits_{x\to \infty} f(x)$

73. $\lim\limits_{x\to 2^+} f(x)$

74. $\lim\limits_{x\to 0} f(x)$

75. $\lim\limits_{x\to -\infty} f(x)$

76. $\lim\limits_{x\to -5} f(x)$

Evaluate. Show supporting work for each problem (algebraic steps or sketch). <u>No calculator</u>.

77. $\lim\limits_{x\to-3}\dfrac{x^2+x-6}{x+3}$

78. $\lim\limits_{x\to0}\dfrac{(x-5)^2-25}{x}$

79. $\lim\limits_{x\to0}\dfrac{\sqrt{x+1}-1}{x}$

80. $\lim\limits_{x\to-6}\dfrac{x+6}{x^2+3x-18}$

81. $\lim\limits_{x\to-2}\dfrac{x^3+8}{x+2}$

82. $\lim\limits_{x\to\infty}\dfrac{3x-5x^2}{4x^2+1}$

83. $\lim\limits_{x\to3^+}\dfrac{1}{x-3}$

84. $\lim\limits_{x\to3^-}\dfrac{1}{x-3}$

85. $\lim\limits_{x\to3}\dfrac{1}{x-3}$

86. $\lim\limits_{x\to3}\dfrac{1}{(x-3)^2}$

87. $\lim\limits_{x\to3^+}[\![x-1]\!]$

88. $\lim\limits_{x\to3^-}[\![x-1]\!]$

89. $f(x)=\begin{cases}1-x, & x\le 1\\ x^2, & x>1\end{cases}$ (a) $\lim_{x\to 1^-} f(x)$ (b) $\lim_{x\to 1^+} f(x)$ (c) $\lim_{x\to 1} f(x)$

90. $f(x)=\begin{cases}\dfrac{x^2-x-6}{x-3} & \text{if } x\ne 3\\ 4 & \text{if } x=3\end{cases}$ (a) $\lim_{x\to 3} f(x)$ (b) $f(3)$

Use the **definition of the derivative** to find the derivative. <u>No calculator</u>.

Definition of the derivative: $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$. (You must **<u>know</u>** this formula.)

91. $f(x)=x^2-8x$

92. $f(x)=\sqrt{x+9}$

93. $f(x)=\dfrac{3}{x-4}$

94. $f(x)=x^3+2x^2-x+4$

Find the **slope of the line tangent to the curve** at the point P by using the formula. <u>No calculator</u>.

Alternate form of def. of derivative: $f'(x)=\lim_{x\to c}\dfrac{f(x)-f(c)}{x-c}$. (You must **<u>know</u>** this formula.)

95. $f(x)=x^2-5x+4,\ P(2,-2)$

96. $f(x)=\sqrt{x+6},\ P(3,3)$

97. $f(x)=x^3+2x^2+1,\ P(-2,1)$

98. $f(x)=2x^2-7x+3,\ P(c,f(c))$

99. Find an equation of a line that is tangent to the curve $f(x)=x^3$ and is parallel to the line $12x-y+10=0$.

100. Find an equation of a line that is tangent to the curve $g(x)=24-x^2$ and is perpendicular to the line $x-8y+6=0$.